

Lepton pair emission in the top quark decay $t \rightarrow bW^+\ell^-\ell^+$

Néstor Quintero,^{1,*} J. Lorenzo Díaz-Cruz,^{1,2,†} and Gabriel López Castro^{1,‡}

¹*Departamento de Física, Centro de Investigación y de Estudios Avanzados,
Apartado Postal 14-740, 07000 México D.F., México*

²*Facultad de Ciencias Físico Matemáticas,
Universidad Autónoma de Puebla, Apartado Postal 1152, Puebla, México*

Abstract

The heaviness of the top quark makes its 2-body Cabibbo-favored decay mode $t \rightarrow bW^+$ to be dominant, at such level that hardly any other decay mode reaches a detectable branching ratio (BR) within the SM. Here we study the decay $t \rightarrow bW^+\ell^-\ell^+$ ($\ell = e, \mu, \tau$), which diverges for massless leptons, and it can reach a BR $\sim \mathcal{O}(10^{-5} \sim 10^{-6})$ for reasonable values of the low energy cut in the lepton-pair invariant mass. This rate surpasses almost any other rare decays such as $t \rightarrow cX$ ($X = \gamma, Z, g, H, W^+W^-$), and thus offers the possibility of being detectable. Furthermore, the estimate of this channel is relevant because it can mimic the signal arising from the lepton number violating decay $t \rightarrow bW^-\ell^+\ell^+$, when the W boson decays into lepton channels.

PACS numbers: 12.15.Ji, 14.60.-z14.65.Ha, 13.40.Ks

* nquintero@fis.cinvestav.mx

† jldiaz@cfm.buap.mx

‡ glopez@fis.cinvestav.mx

I. INTRODUCTION

From our current explorations on the high energy frontier, we know that the top quark stands as the heaviest particle within the Standard Model (SM), which has been considered somehow peculiar and a hint that could help us to understand the nature of electroweak symmetry breaking. It is thus very important to study the top quark properties in order to search for these connections. In this regard, it is known that the top is also the only fermion massive enough to undergo first order weak decays, such that its dominant decay channel $t \rightarrow bW^+$ has a large decay width Γ_t , of similar size to the one for gauge bosons. This has encouraged the calculation of higher-order corrections to the rate of this decay channel: the first- [1, 2] and second-order [3] QCD corrections, the first order electroweak corrections [2, 4] and the finite W boson width effects [1, 5] have been reported in the last twenty years. As it was summarized in ref. [3], these corrections turn out to be (in percent) -8.58 , -2.09 , $+1.69$ and -1.49 for $m_t = 173.5$ GeV, respectively¹.

The dominance of the 2-body decay mode $t \rightarrow bW^+$ suppresses considerably any other decay channel, making them hardly detectable. These include the decay modes $t \rightarrow s(d)W^+$, which contribute altogether less than one per mille to Γ_t . In the case of 3-body decay modes, such as $t \rightarrow bW^+(\gamma, g)$ [6–11], $t \rightarrow bW^+Z$ [9–15], and $t \rightarrow bW^+H$ [9–11, 14, 16, 17], the corresponding branching fractions (BR) are even smaller. Similarly, the FCNC modes $t \rightarrow cX$ ($X = \gamma, g, Z, H$) [18–21], $t \rightarrow cW^+W^-(ZZ, \gamma\gamma)$ [13, 22–24] and $t \rightarrow c\ell^-\ell^+$ [25] have an extremely suppressed BR, although some of them involve very interesting dynamical mechanisms and have been suggested as possible probes of New Physics (NP), see for instance [26, 27], which produces an enhancement on the BR that could make them detectable. For the kinematically suppressed decay channels, the subsequent decay of unstable bosons is understood and sometimes have been taken into account.

In this paper we study the 4-body top quark decays $t \rightarrow bW^+\ell^-\ell^+$ ($\ell = e, \mu$ or τ). The rates of these decays are of order α^2 with respect to $t \rightarrow bW$ and have a divergent behavior for massless leptons due to the k^{-2} dependence of the photon propagator. For light leptons (see [28] for a related discussion in the case of the μ lifetime), this rate should be included in the definition of the top quark width in order to cancel the large QED corrections of order $\mathcal{O}(\alpha^2)$ in $t \rightarrow bW^+$, arising from a light lepton loop-insertion in the infrared photon propagator. Here, we are also interested in their study because they could mimic the signal from $t \rightarrow bW^-\ell^+\ell'^+$ when the W bosons are detected through its leptonic decays. The later is a lepton number ($\Delta L = 2$) violating decay, which has been suggested as a signal of NP, induced by heavy Majorana neutrinos or doubly charged Higgs exchange [29]. Some aspects related to the order α behavior of the radiative ($t \rightarrow bW^+\gamma$) and lepton-pair production ($t \rightarrow bW^+\ell^-\ell^+$) decay rates are discussed too. The top decay channel under consideration has not been widely studied before. To the best of our knowledge, only in reference [30] it has been suggested that $t \rightarrow bW^+Z'$, with a subsequent decay of the Z' boson into a lepton pair, may be a useful mechanism to detect a dark light Z' gauge boson [31].

The organization of this work goes as follows. In section II we briefly recount the radiative decay $t \rightarrow bW^+\gamma$, in order to define notation and express the amplitude in such a way that

¹ In numerical evaluations we use this value of the top quark mass

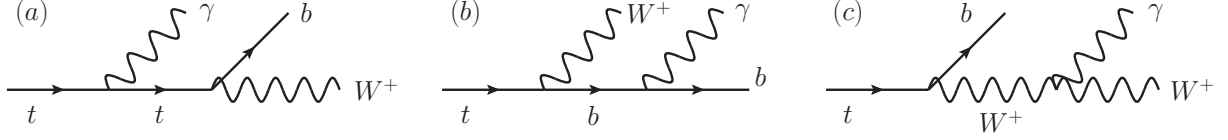


FIG. 1. Feynman diagrams that contribute to $t \rightarrow bW^+\gamma$.

is simple to identify the gauge invariance of the full amplitude. In section III we present the 4-body decay $t \rightarrow bW^+\ell^-\ell^+$, discussing in detail the dependence in calculation of the IR cutoff. Conclusions are left for section IV.

II. RADIATIVE TOP QUARK DECAY $t \rightarrow bW^+\gamma$

The radiative top quark (3-body) decay $t(p_t) \rightarrow b(p_b)W^+(p_W)\gamma(k)$ has been widely considered in previous works [6–11]. We briefly discuss its amplitude, which is written in a form that is convenient to compare with the 4-body channel (sec. III); the numerical result for the decay rate is also included in order to discuss the infrared (IR) behavior. The Feynman diagrams that contribute in the unitary gauge are shown in Fig. 1. Thus, the total decay amplitude is written as follows

$$\begin{aligned} \mathcal{M}_{\text{top}}^\gamma &\equiv \mathcal{M}(t \rightarrow bW^+\gamma), \\ &= \left(\frac{-ig e}{\sqrt{2}}\right) V_{tb} \varepsilon_\mu^W \varepsilon_\nu^\gamma \bar{u}(p_b) \left[Q_t \mathcal{T}_t^{\mu\nu}(k^2) + Q_b \mathcal{T}_b^{\mu\nu}(k^2) + Q_W \mathcal{T}_W^{\mu\nu}(k^2) \right] u(p_t), \end{aligned} \quad (1)$$

where

$$\mathcal{T}_t^{\mu\nu}(k^2) = \gamma^\mu P_L \frac{(\not{p}_t - \not{k}) + m_t}{k^2 - 2p_t \cdot k} \gamma^\nu, \quad (2)$$

$$\mathcal{T}_b^{\mu\nu}(k^2) = \gamma^\nu \frac{(\not{p}_b + \not{k}) + m_b}{k^2 + 2p_b \cdot k} \gamma^\mu P_L, \quad (3)$$

$$\mathcal{T}_W^{\mu\nu}(k^2) = \gamma^\alpha P_L \Delta_{\alpha\beta}^W(h) \Gamma_{WW\gamma}^{\beta\mu\nu}. \quad (4)$$

The amplitude for a real photon emission is obtained by taking $k^2 = 0$ in the above expressions. P_L denotes the left-handed chiral projector, (Q_b, Q_t, Q_W) are the particles electric charges in units of e , and ε_μ^W (ε_ν^γ) is the four-vector polarization of the W boson (photon), respectively. The CKM quark mixing matrix element is taken as $V_{tb} = 1$. The propagator of the W boson in the unitary gauge is denoted by $\Delta_{\alpha\beta}^W(h) = (-g_{\alpha\beta} + h_\alpha h_\beta / M_W^2) / (h^2 - M_W^2)$, with $h = p_W + k$, while the triple gauge boson vertex, with our assignment of momenta $W(p_W + k) \rightarrow W(p_W)\gamma(k)$, is given by

$$\Gamma_{WW\gamma}^{\beta\mu\nu} = (p_W - k)^\beta g^{\mu\nu} - (2p_W + k)^\nu g^{\mu\beta} + (2k + p_W)^\mu g^{\nu\beta}. \quad (5)$$

The decay amplitude $\mathcal{M}_{\text{top}}^\gamma$ is gauge-invariant owing to the charge conservation condition $Q_b - Q_t + Q_W = 0$ [7] and diverges in the soft-photon limit ($k \rightarrow 0$). The photon energy

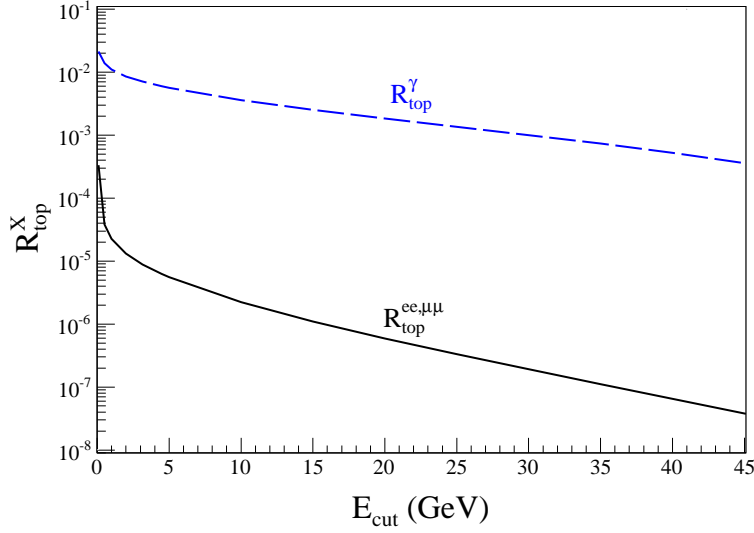


FIG. 2. Ratios R_{top}^X as a function of E_{cut} for the radiative channel ($X = \gamma$) and lepton-pair channels ($X = e^-e^+, \mu^-\mu^+$).

spectrum is obtained by integrating the unpolarized squared amplitude over $s_1 = (p_b + k)^2$, namely:

$$\frac{d\Gamma(t \rightarrow bW^+\gamma)}{dE_\gamma} = \frac{1}{128\pi^3 m_t^2} \int_{s_1^-}^{s_1^+} ds_1 |\mathcal{M}_{\text{top}}^\gamma|^2. \quad (6)$$

For completeness we show here the integration limits:

$$s_1^\pm(s_2) = M_W^2 + \frac{m_t^2 - s_2}{2s_2} \left[(s_2 - m_b^2 + M_W^2) \pm \sqrt{\lambda(s_2, M_W^2, m_b^2)} \right], \quad (7)$$

with $s_2 = m_t^2 - 2m_t E_\gamma$ and $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$. The photon energy goes from 0 up to a maximum value $E_\gamma^{\text{max}} = [m_t^2 - (M_W + m_b)^2]/2m_t$. However, because of the IR divergence, it is necessary to impose a cut in the minimum photon energy ($E_{\gamma, \text{cut}}$) in order to obtain a finite value.

The normalized radiative rate is defined as $R_{\text{top}}^\gamma \equiv \Gamma(t \rightarrow bW^+\gamma)/\Gamma(t \rightarrow bW^+)$, where the 2-body decay width at leading order is given by [32]

$$\Gamma(t \rightarrow bW^+) = \frac{G_F m_t^3}{8\pi\sqrt{2}} (1 - x_t^2)^2 (1 + 2x_t^2), \quad (8)$$

with $x_t = M_W/m_t$. R_{top}^γ is plotted in Fig. 2 (dashed line) as a function of $E_{\gamma, \text{cut}}$. Our results are consistent with those obtained in Refs. [6–11]. It is well known that the soft-photon divergence in R_{top}^γ will be cancelled by the corresponding infrared divergence from the one-loop virtual photon corrections, giving rise to a finite photon inclusive rate $\Gamma(t \rightarrow bW^+(\gamma))$. As it can be seen from Fig. 2, the expected $\mathcal{O}(\alpha)$ suppression of the radiative decay, with respect to the dominant 2-body decay mode, is already evident for $E_{\gamma, \text{cut}} \simeq 3$ GeV.

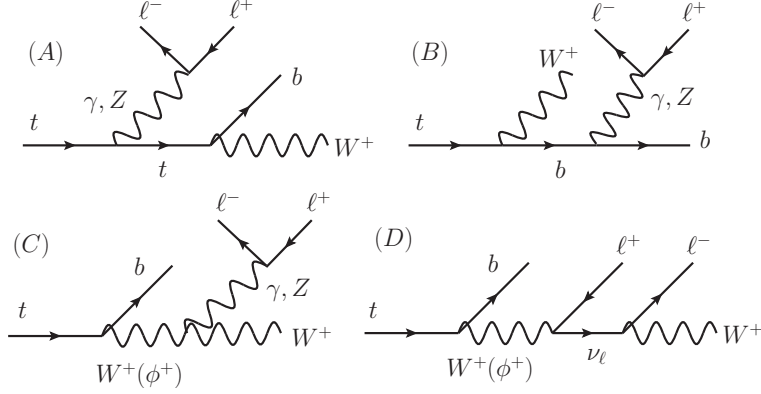


FIG. 3. Feynman diagrams that contribute to $t \rightarrow bW^+\ell^-\ell^+$.

III. FOUR-BODY DECAY $t \rightarrow bW^+\ell^-\ell^+$

Now, let us consider the lepton-pair production in top quark decays $t(p_t) \rightarrow b(p_b)W^+(p_W)\ell^-(p_1)\ell^+(p_2)$ with $\ell = e, \mu$ or τ . The relevant diagrams for this decay are shown in Fig. 3. The diagrams Fig. 3(A)-(C) are generated by demanding that the virtual photon (Z boson) converts into a lepton-pair². An additional contribution induced by a neutrino exchange is shown in Fig. 3(D). The full set of diagrams is required in order to fulfill independence upon the electroweak gauge parameters.

The dominant photon-exchange contribution to the amplitude is written as

$$\begin{aligned} \mathcal{M}_{\text{top}}^{\ell\ell} &\equiv \mathcal{M}(t \rightarrow bW^+\ell^-\ell^+), \\ &= \left(\frac{-ige}{\sqrt{2}}\right) V_{tb} \varepsilon_\mu^W \ell_\nu \bar{u}(p_b) \left[Q_t \mathcal{T}_t^{\mu\nu}(k^2) + Q_b \mathcal{T}_b^{\mu\nu}(k^2) + Q_W \mathcal{T}_W^{\mu\nu}(k^2) \right] u(p_t), \end{aligned} \quad (9)$$

where $\ell_\nu = e[\bar{u}(p_1)\gamma_\nu v(p_2)]/k^2$, with $k = p_1 + p_2$ the virtual photon momentum. The $\mathcal{T}_i^{\mu\nu}(k^2)$ tensors are given in Eqs. (2)-(4), this time keeping $k^2 \neq 0$ in the propagators. In fact, the amplitude $\mathcal{M}_{\text{top}}^{\ell\ell}$, Eq. (9), is easily obtained from Eq. (1) by replacing the photon polarization (ε_ν^γ) by the effective leptonic current (ℓ_ν).

The contributions of the Z -boson exchange amplitude (Fig. 3A-C) are suppressed owing to the large mass of this gauge boson. An estimate of its largest contribution can be obtained when the Z boson is taken on-shell. In this case the branching ratio for $t \rightarrow bW^+\ell^-\ell^+$ can be estimated from the SM prediction for $\Gamma(t \rightarrow bW^+Z)/\Gamma(t \rightarrow bW^+) \simeq 2 \times 10^{-6}$ [12] followed by the leptonic decay of the Z boson, which yields:

$$\frac{\Gamma(t \rightarrow bW^+Z) \times \mathcal{B}(Z \rightarrow \ell^-\ell^+)}{\Gamma(t \rightarrow bW^+)} \simeq 6 \times 10^{-8}. \quad (10)$$

As it will be shown, this is smaller than the photon contribution. Finally, the neutrino exchange diagram (Fig. 3(D)) is necessary to guarantee a gauge-invariant result for the

² The lepton-pair can also be produced by an intermediate Higgs boson, however this gives a negligible contribution.

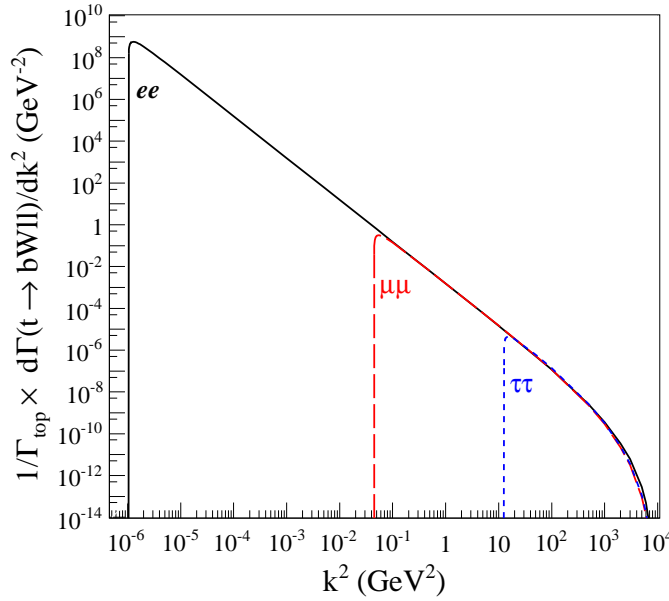


FIG. 4. Normalized invariant-mass distribution for e^-e^+ (solid line), $\mu^-\mu^+$ (long-dashed line) and $\tau^-\tau^+$ (short-dashed line) channels.

full decay amplitude [12], yet its contribution is even smaller than the one from Z boson exchange. Thus, the leading contribution is given by Eq. (9).

In Figure 4 we plot the invariant mass distribution of the lepton pair for the three lepton flavors. This observable is peaked close to the threshold for lepton pair production owing to the $1/k^2$ dependence of the photon propagator from the decay amplitude. As it can be checked, this spectrum diverges in the soft limit for massless leptons ($k^2 \rightarrow 0$); this divergence would be eventually cancelled by the corresponding IR divergence resulting from the QED corrections to $t \rightarrow bW^+$ decay at two-loops, specifically from the massless lepton loop insertion to the photon propagators. In practice, since electrons and muons have a small (but nonzero) mass, the branching ratio may become meaninglessly large when integrating over the full range of k^2 , making necessary the introduction of an infrared cutoff k_{cut}^2 .

Given the very steeply behavior of the squared amplitude close to the lepton pair threshold, it is convenient to check the stability of our numerical result. This is done by evaluating the ratio $R_{\text{top}}^{\ell\ell} \equiv \Gamma(t \rightarrow bW^+\ell^-\ell^+)/\Gamma(t \rightarrow bW^+)$ (with $\ell = e, \mu, \tau$) using two numerical methods: (i) integrating over the lepton-pair invariant mass distribution and, (ii) by integrating upon the five independent kinematical variables of the four-body phase space. We have checked that both methods give identical results, except when the integration over k^2 is extended until the lepton-pair threshold in the case of electrons. The $R_{\text{top}}^{ee,\mu\mu}$ ratios as a function of k_{cut}^2 are shown in Table I. As previously advertised, the ratio R_{top}^{ee} for producing an e^-e^+ pair becomes larger than unity for $k_{\text{cut}}^2 < 10^{-3} \text{ GeV}^2$. Although for lepton pairs $\mu^-\mu^+$ and $\tau^-\tau^+$ the obtained value is not greater than one, the result in the case of muons is still unexpectedly large from the perturbative point of view. However, as we can see from Table I, in all cases the expected behavior for the branching ratio, of $O(10^{-5} \sim 10^{-6})$ as

TABLE I. The ratio $R_{\text{top}}^{\ell\ell}$ ($\ell = e, \mu, \tau$) as a function of k_{cut}^2 . The instability for the result in the first entry of R_{top}^{ee} is indicated with a star symbol

k_{cut}^2 (GeV ²)	R_{top}^{ee}	$R_{\text{top}}^{\mu\mu}$	$R_{\text{top}}^{\tau\tau}$
$4m_\ell^2$	6.49*	1.52×10^{-2}	1.06×10^{-5}
10^{-3}	0.85	—	—
10^{-2}	8.52×10^{-2}	—	—
10^{-1}	8.48×10^{-3}	8.48×10^{-3}	—
1	8.28×10^{-4}	8.28×10^{-4}	—
10	7.40×10^{-5}	7.40×10^{-5}	—
20	6.31×10^{-6}	6.31×10^{-6}	9.15×10^{-6}
50	3.70×10^{-6}	3.70×10^{-6}	5.50×10^{-6}

determined by the $\mathcal{O}(\alpha^2)$ of the decay rates, can be obtained for values of $k_{\text{cut}}^2 \geq 20$ GeV².

In Figure 2, we compare the IR divergent behavior of the normalized rates for $t \rightarrow bW^+e^-e^+(\mu^-\mu^+)$ (solid-line) and $t \rightarrow bW^+\gamma$ (dashed-line) as a function of E_{cut} . By notation, in the case of the radiative decay $E_{\text{cut}} = E_{\gamma, \text{cut}}$ corresponds to the photon energy, while for the four-body channel $E_{\text{cut}} = \sqrt{k_{\text{cut}}^2}$ is the squared root of the invariant mass for the lepton pair. We observe that these ratios are of order $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha)$, respectively, for relatively low values of the corresponding IR cutoffs (of the order $E_{\text{cut}} \simeq 3$ GeV). Above this critical value, the ratio between these two decay modes becomes of order $\mathcal{O}(\alpha)$ as it would be expected.

IV. CONCLUSIONS

After the success of the first stage of the LHC, with the detection of the Higgs boson as its shining trophy, we hope that the next one, with higher energy, will continue to test the SM and hopefully find a signal of new physics. In particular, the LHC will also become a productive top factory, which will be able to test the top properties, its couplings to SM channels and rare decays. Having about $10^7 - 10^8$ top pairs produced per year at LHC, it is expected that rare decays with BR of order $10^{-5} - 10^{-6}$ may be detectable, depending on the signal.

Along these lines, we have presented in this paper the (first) calculation of the process where a lepton pair is emitted in top quark decays. Both, the spectrum and the branching ratios exhibit an infrared divergent behaviour in the limit of massless leptons. We observed that by cutting at $\ell^-\ell^+$ invariant masses larger than 20 GeV², the branching ratios follow the suppression rule expected by their relative order in α^2 .

The lepton pair production processes $t \rightarrow bW^+(\rightarrow \ell'^+\nu_\ell)\ell^+\ell^-$ can provide an important background for searches of lepton number violating (LNV) top quark decays $t \rightarrow bW^-(\rightarrow \ell^-\bar{\nu})\ell^+\ell'^+$, when W bosons are detected through lepton channels. Table I shows that even by

cutting the large phase space in the lepton-pair invariant mass, the fraction of conventional decays is larger than the LNV decays, whose most optimistic estimates are of order a few $\times 10^{-6}$ [29].

Acknowledgements: The authors would like to thank Conacyt (México) for financial support. They are also grateful to A. Guevara and E. Camacho for useful conversations.

-
- [1] M. Jezabek and J. H. Kuhn, Nucl. Phys. **B314**, 1 (1989); Phys. Rev. D **48**, R1910 (1993); A. Czarnecki, Phys. Lett. **B252**, 467 (1990); C. S. Li, R. J. Oakes and T. C. Yuan, Phys. Rev. D **43**, 3759 (1991).
 - [2] A. Denner and T. Sack, Nucl. Phys. **B358**, 46 (1991).
 - [3] A. Czarnecki and K. Melnikov, Nucl. Phys. **B544**, 520 (1999); J. Gao, C. S. Li and H. X. Zhu, Phys. Rev. Lett. **110**, 042001 (2013).
 - [4] G. Eilam, R. R. Mendel, R. Migneron, and A. Soni, Phys. Rev. Lett. **66**, 3105 (1991); J. -A. Liu and Y. -P. Yao, Int. J. Mod. Phys. A **6**, 4925 (1991).
 - [5] G. Calderón and G. López Castro, Int. J. Mod. Phys. A **23**, 3525 (2008) [hep-ph/0108088].
 - [6] G. Tupper, J. Reid, G. Li, and M. Samuel, Phys. Rev. D **43**, 274 (1991)
 - [7] G. Couture, Phys. Rev. D **40**, 2927 (1989); **42**, 1855(E) (1990).
 - [8] V. Barger, A. Stange, and W.-Y. Keung, Phys. Rev. D **42**, 1835 (1990).
 - [9] V. Barger, W.-Y. Keung, and T. G. Rizzo, Phys. Rev. D **40**, 2274 (1989).
 - [10] G. Mahlon, hep-ph/9810485.
 - [11] R. Decker, M. Nowakowski, and A. Pilaftsis, Z. Phys. C **57**, 339 (1993) [hep-ph/9301283].
 - [12] G. Altarelli, L. Conti, and V. Lubicz, Phys. Lett. B **502**, 125 (2001) [hep-ph/0010090].
 - [13] E. Jenkins, Phys. Rev. D **56**, 458 (1997) [hep-ph/9612211].
 - [14] G. Mahlon and S. Parke, Phys. Lett. B **347**, 394 (1995) [hep-ph/9412250].
 - [15] J. L. Díaz Cruz and D. A. López Falcon, Phys. Rev. D **61**, 051701 (2000) [hep-ph/9911407].
 - [16] T. G. Rizzo, Phys. Rev. D **35**, 1067 (1987), V. D. Barger and W.-Y. Keung, Phys. Lett. B **202**, 393 (1988).
 - [17] T. Han and R. Ruiz, Phys. Rev. D **89**, 074045 (2014) arXiv:1312.3324 [hep-ph].
 - [18] J. L. Díaz-Cruz, R. Martinez, M. A. Perez and A. Rosado, Phys. Rev. D **41**, 891 (1990).
 - [19] G. Eilam, J. L. Hewett and A. Soni, Phys. Rev. D **44**, 1473 (1991) [Erratum-ibid. D **59**, 039901 (1999)].
 - [20] B. Mele, S. Petrarca, and A. Soddu, Phys. Lett. B **435**, 401 (1998) [hep-ph/9805498].
 - [21] J. A. Aguilar-Saavedra and B. M. Nobre, Phys. Lett. B **553**, 251 (2003) [hep-ph/0210360].
 - [22] S. Bar-Shalom, G. Eilam, A. Soni, and J. Wudka, Phys. Rev. Lett. **79**, 1217 (1997) [hep-ph/9703221]; Phys. Rev. D **57**, 2957 (1998) [hep-ph/9708358].
 - [23] S. Bar-Shalom, G. Eilam, M. Frank, and I. Turan, Phys. Rev. D **72**, 055018 (2005) [hep-ph/0506167].
 - [24] J. L. Díaz-Cruz, M. A. Perez, G. Tavares-Velasco, and J. J. Toscano, Phys. Rev. D **60** 115014 (1999) [hep-ph/9903299].
 - [25] M. Frank and I. Turan, Phys. Rev. D **74**, 073014 (2006) [hep-ph/0609069];

- [26] J. A. Aguilar-Saavedra, Acta Phys. Polon. B **35**, 2695 (2004) [[hep-ph/0409342](#)].
- [27] F. Larios, R. Martínez, and M. A. Pérez, Int. J. Mod. Phys. A **21**, 3437 (2006) [[hep-ph/0605003](#)].
- [28] T. van Ritbergen and R. G. Stuart, Phys. Rev. Lett. **82**, 488 (1999); Nucl. Phys. B **564**, 343 (2000).
- [29] S. Bar-Shalom, N. Deshpande, G. Eilam, J. Jiang, and A. Soni, Phys. Lett. B **643**, 242 (2006) [[hep-ph/0608309](#)]; Z. Si and K. Wang, Phys. Rev. D **79**, 014034 (2009); N. Quintero, G. López Castro and D. Delepine, Phys. Rev. D **84**, 096011 (2011) [Erratum-ibid. D **86**, 079905 (2012)] [[arXiv:1108.6009](#) [hep-ph](#)]; N. Quintero, Phys. Rev. D **87**, 056005 (2013) [[arXiv:1212.3016](#) [hep-ph](#)].
- [30] K. Kong, H. -S. Lee and M. Park, Phys. Rev. D **89**, 074007 (2014) [[arXiv:1401.5020](#) [hep-ph](#)].
- [31] R. Essig, J. A. Jaros, W. Wester, P. H. Adrian, S. Andreas, T. Averett, O. Baker and B. Batell *et al.*, [[arXiv:1311.0029](#) [hep-ph](#)].
- [32] J. Beringer *et al.* [Particle Data Group], Phys. Rev. D **86**, 010001 (2012).